Surface-integral formalism for transfer reactions:

Applications and applicability for (d,p) reactions

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Studying nuclear structure with \((d,p)\) one-nucleon transfers

\((d,p)\) reactions:
- Simplest mechanism for adding a neutron
- Traditionally used to study stable nuclei
- Used in inverse kinematics at RIB facilities, for studying weakly-bound systems

Theoretical descriptions of \((d,p)\) reactions:
- Progress over the years: Plane-wave theory, DWBA (zero-range & finite-range), coupled-channels approach, breakup, etc.

But: Current theories of \((d,p)\) reactions not very useful for transfers to resonance states:
- Conceptual: extracting spectroscopic information
- Practical: convergence issues
Resonances in low-energy nuclear physics

Resonances:
• Unstable quantum-mechanical states
• Occur in light, medium-mass, and heavy nuclei
• Crucially affect astrophysical reaction rates
• Abundant in weakly-bound nuclei

Current approach:
• Apply standard (d,p) descriptions to resonances
• Increase model space to achieve convergence

Evolution of single-particle energies and the location of the neutron drip line

$^{20}\text{O}(d,p)^{21}\text{O}$ inverse-kinematics experiment at GANIL to determine N=16 shell gap

Otsuka et al, PRL 105, 032501 (2010)

Fernandez-Dominguez et al, PRC 84, 011301 (R) (2011)
Describing resonances in binary reactions

**R-matrix approach:**
- **Main idea:** divide space into 2 regions:
  - \( r \leq a \) -- interior: nuclear and Coulomb interactions
  - \( r > a \) -- exterior: Coulomb only
- **Formalism:**
  - Interior: expand nuclear wave function in set of basis functions
  - Exterior: scattering wave function
  - Surface: matching conditions allow to parameterize collision matrix \( \Rightarrow \) expressions for cross sections
- **Connect observed parameters** \((E_R, \Gamma)\) to formal parameters \((\hat{E}_R, \gamma^2)\)
- **Typical applications** adjust parameters to reproduce measured cross sections

**Experimental studies of resonances:**
- Elastic & inelastic scattering, capture, etc.

**Characterization of resonances:** position & widths

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**Kopecky and Plompen, JRC Report (2011)**

**Satchler, Introduction to Nuclear Reactions (1980)**
Exploring R-matrix ideas for (d,p) one-nucleon transfers

Proposed new formalism (Mukhamedzhanov, 2011):

- R-matrix concepts:
  - surface separating internal and external regions
  - cross sections expressed in terms of reduced widths, logarithmic derivatives, surface radii

- Goals for (d,p):
  - useful for resonances
  - reduce dependence on model for interior
  - extract useful spectroscopic quantities from comparison to experiment (widths)

- Formalism:
  - applicable to stripping to bound and resonance states
  - general enough to include deuteron breakup contributions via CDCC (continuum-discretized coupled-channels method)
  - bonus: resolves practical issues related to numerical convergence

Formalism:
Mukhamedzhanov, PRC 84, 044616 (2011)
Exploring R-matrix ideas for (d,p) one-nucleon transfers II

Transition matrix element $M$:
- Connects initial to final wave function
- Cross section $\sigma \sim M^2$

$$M^{(\text{post})} = \langle \Phi_f(-) | \Delta V_{pF} | \Psi_i(+) \rangle$$

**DWBA**

$$\langle \varphi_F \chi_{pF}(-) | \Delta V_{pF} | \varphi_d \varphi_A \chi_{dA}(+) \rangle$$

$$\langle I_A^F \chi_{pF}(-) | \Delta V_{pF} | \varphi_d \chi_{dA}(+) \rangle$$

$\Psi_i(+) :$ exact d+A scattering function
$\Phi_f(-) = \varphi_F \chi_{pF}(-)$ exit channel function
$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$
$I_A^F = \langle \varphi_A | \varphi_F \rangle$

**3-body**

$$\langle \varphi_F \chi_{pF}(-) | \Delta V_{pF} | \varphi_A \Psi_i^{3B(+) \rangle}$$

$$\langle I_A^F \chi_{pF}(-) | \Delta V_{pF} | \Psi_i^{3B(+) \rangle}$$

**CDCC**

$$\langle I_A^F \chi_{pF}(-) | \Delta V_{pF} | \Psi_i^{\text{CDCC}(+) \rangle}$$

**One-body overlap of A and A+1 systems**
- carries structure information
- typically approximated by single-particle function

$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$
Generalized R-matrix formalism for (d,p) reactions

Splitting the transition matrix element $M$:
- Interior and exterior with respect to $r_{nA}$

$M^{(post)} = < \Phi_f^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} >$

**Interior + exterior**

$M^{(post)} = M^{(post)}(0,a) + M^{(post)}(a,\infty)$

$I_A^F = < \Phi_A | \Phi_F > = I_A^F(r_{nA})$

**Mukhamedzhanov**

$M^{(post)}(a,\infty) = M_{surf}(a) + M^{(prior)}(a,\infty)$

$M_{surf}(a) = < I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \Phi_d \chi_{dA}^{(+)} >_{ext}$

$$
\int_{r \leq R} dr f(r) [\nabla \cdot \mathbf{T} - \mathbf{T} \cdot \nabla] g(r)
$$

$$
= -\frac{1}{2\mu} \int_{r=R} dS [g(r) \nabla f(r) - f(r) \nabla g(r)]
$$

$$
= -\frac{1}{2\mu} R^2 \int d\Omega_r \left[ g(r) \frac{\partial f(r)}{\partial r} - f(r) \frac{\partial g(r)}{\partial r} \right]_{r=R}
$$

**Surface term**

$M_{surf}(a) = f(a, C_A^F, b_{nA})$

$b_{nA} = \log \text{ derivative of } I_A^F \text{ at surface radius } a$

ANC: $C_A^F$ defined through: $I_A^F(r_{nA}) \rightarrow C_A^F W(kr_{nA})$

related to reduced width amplitude $C_A^F \sim \gamma_{nA}$
Generalized R-matrix formalism for (d,p) reactions II

**Assessing the approach:**
- Internal – external separation sensible?
- Dominant surface term? Size of corrections?
- Study cross sections arising from different terms
- Start with DWBA and bound states
- Investigate resonances

**Cases considered so far:**
- $^{90}\text{Zr}(d,p)$ for $E_d=11$ MeV
  - $^{91}\text{Zr}$ gs, 1st excited state, 2f$_{7/2}$ resonance
- $^{48}\text{Ca}(d,p)$ for $E_d=13$ MeV
  - $^{49}\text{Ca}$ gs, 1st excited state
- $^{20}\text{O}(d,p)$ for $E_d=21$ MeV
  - $^{21}\text{O}$ gs, 1st excited state, 1d$_{5/2}$ and 1f$_{7/2}$ resonances
- $^{12}\text{C}(d,p)$ for $E_d=30$ MeV
- $^{40}\text{Ca}(d,p)$ for $E_d=34.4$ MeV
- $^{209}\text{Pb}(d,p)$ for $E_d=52$ MeV
- Planned: $^{48}\text{Ca}(d,p)$ for $E_d=19.3$ and 56 MeV

**DWBA matrix element**

$$M^{(\text{post})} = M^{(\text{post})}(0,a) + M^{(\text{surf})}(a) + M^{(\text{prior})}(a,\infty)$$

$$M^{(\text{surf})}(a) = \sqrt{\frac{R_{nA}}{2\mu_{nA}}} \sum_{J_{nA}M_{nA}j_{nA}m_{nA}j_{mA}m_{mA}} \langle J_{nA}M_{nA}j_{nA}m_{nA} | J_{F}M_{F} \rangle \langle J_{nA}M_{nA}l_{nA}m_{l_{nA}} | j_{nA}m_{j_{nA}} \rangle \langle J_{p}M_{p}j_{nA}m_{j_{nA}} | J_{d}M_{d} \rangle \frac{\partial \varphi_{d}(r_{pA})}{\partial r_{nA}}$$
Assessing the R-matrix ideas Ia

1. Interior vs exterior contributions

\[ M = M(0,a) + M(a,\infty) \]

This case:

- \(^{90}\text{Zr}(d,p)\) for \(E_d=11\) MeV
- \(^{91}\text{Zr}\) gs \((5/2^+ )\)
  - 1\(^{st}\) excited state \((1/2^+ )\)
  - \(2f_{7/2}\) resonance

Observations

- ‘action is in the nuclear surface’
- Post formalism more sensitive to larger radii than prior:

\[
M^{(\text{post})} = \langle \Phi_i^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} \rangle \\
\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF} \\
M^{(\text{prior})} = \langle \Psi_i^{(-)} | \Delta V_{dA} | \Phi_i^{(+)} \rangle \\
\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}
\]
Assessing the R-matrix ideas lb

1. Interior vs exterior contributions

\[ M = M(0,a) + M(a,\infty) \]

This case:
- \(^{90}\text{Zr}(d,p)\) for \(E_d=11\ \text{MeV}\)
- \(^{91}\text{Zr}\;\text{gs}\; (5/2^+)\)
- 1st excited state \((1/2^+),\ 2f_{7/2}\) resonance

Observations
- ‘action is in the nuclear surface’
- Post formalism more sensitive to larger radii than prior:

\[
M^{(\text{post})} = \langle \Phi_{f}^{(-)} | \Delta V_{pF} | \Psi_{i}^{(+)} \rangle \\
\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}
\]

\[
M^{(\text{prior})} = \langle \Psi_{f}^{(-)} | \Delta V_{dA} | \Phi_{i}^{(+)} \rangle \\
\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}
\]
Assessing the R-matrix ideas Ic

1. Interior vs exterior contributions

\[ M = M(0,a) + M(a,\infty) \]

This case:
- \(^{90}\text{Zr}(d,p)\) for \(E_d=11\ \text{MeV}\)
- \(^{91}\text{Zr}\) gs (5/2+)
- 1st excited state (1/2+)
- \(2f_{7/2}\) resonance

Observations
- ‘action is in the nuclear surface’
- Post formalism more sensitive to larger radii than prior:

\[
M^{(\text{post})} = \langle \Phi_i^{(-)} | \Delta V_{\text{pF}} | \Psi_i^{(+)} \rangle \\
\Delta V_{\text{pF}} = V_{\text{pA}} + V_{\text{pn}} - U_{\text{pF}}
\]

\[
M^{(\text{prior})} = \langle \Psi_i^{(-)} | \Delta V_{\text{dA}} | \Phi_i^{(+)} \rangle \\
\Delta V_{\text{dA}} = V_{\text{pA}} + V_{\text{nA}} - U_{\text{dA}}
\]

- Resonance: reduced contribution from interior, more pronounced surface effect
Assessing the R-matrix ideas IIa

2. Surface contribution

\[ M = M^{(post)}(0,a) + M^{(surf)}(a) + M^{(prior)}(a,\infty) \]

This case:
- \(^{90}\text{Zr}(d,p)\) for \(E_d=11\) MeV
- \(^{91}\text{Zr} gs\) (5/2+)
- 1\(^{st}\) excited state (1/2+)
- 2f_{7/2} resonance

Observations
- Surface term indeed dominant 6-8 fm
- Small interior contributions \(\rightarrow\) little dependence on model for interior
- Small exterior contributions \(\rightarrow\) better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external

Peak cross section relative to full calculation

Angular cross sections for \(a=7\) fm
Assessing the R-matrix ideas IIb

2. Surface contribution

\[ M = M^{(post)}(0,a) + M^{(surf)}(a) + M^{(prior)}(a,\infty) \]

This case:
- \(^{90}\text{Zr}(d,p)\) for \(E_d=11\) MeV
  - \(^{91}\text{Zr}\) gs (5/2+)
  - 1st excited state (1/2+)
  - 2f\(_{7/2}\) resonance

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Peak cross section relative to full calculation

Angular cross sections for \(a=7\) fm
Assessing the R-matrix ideas IIc

2. Surface contribution

\[ M = M^{(\text{post})}(0,a) + M_{\text{(surf)}}(a) + M^{(\text{prior})}(a,\infty) \]

This case:
- \(^{90}\text{Zr}(d,p)\) for \(E_d=11\) MeV
  - \(^{91}\text{Zr}\) gs (5/2+)
  - 1st excited state (1/2+)
  - 2f\(_{7/2}\) resonance

Observations
- Surface term indeed dominant 6-8 fm
- Small interior contributions \(\rightarrow\) little dependence on model for interior
- Small exterior contributions \(\rightarrow\) better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external
- Reduced interior contribution at peak for surface term

Peak cross section relative to full calculation

Angular cross sections for \(a = 8\) fm
Assessing the R-matrix ideas - $^{48}$Ca

2. Surface contribution

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a, \infty)$$

This case:

- $^{48}$Ca(d,p) for $E_d=13$ MeV
  - $^{49}$Ca gs (3/2-)
  - 1st excited state (1/2-)

Observations

- Surface term indeed dominant 5-8 fm
- Small interior contributions $\rightarrow$ little dependence on model for interior
- Small exterior contributions $\rightarrow$ better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external
- Cross section calculated from surface term sensitive to surface radius
Assessing the R-matrix ideas - $^{48}$Ca

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$$M = M^{(post)}(0, a) + M_{(surf)}(a) + M^{(prior)}(a, \infty)$$

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- Cross section calculated from surface term sensitive to surface radius

This case:

- $^{48}$Ca(d,p) for $E_d=13$ MeV
- $^{49}$Ca gs (3/2-)
- 1st excited state (1/2-)

Peak cross section relative to full calculation

Surface angular cross sections for various a values
Assessing the R-matrix ideas – $^{20}$O

In progress
Data for both bound and resonance states available!

This case:
- $^{20}$O(d,p) for $E_d=21$ MeV
  - $^{20}$O gs (3/2-)
  - 1st excited state (1/2-)
  - 1d$_{3/2}$ and 1f$_{7/2}$ resonances

Cross sections for bound states compared to data

Phase shifts for resonances considered
Next: Extension of the formalism to include breakup

CDCC (Continuum-discretized coupled channels)
- Approximate treatment of 3-body problem
- Describes breakup of deuteron
- Successfully used for describing data
- Currently revisited via comparison with Fadeev

**DWBA matrix element**

\[ M^{(\text{post})} = M^{(\text{post})}(0,a) + M^{(\text{surf})}(a) + M^{(\text{prior})}(a,\infty) \]

**CDCC matrix element**

\[ M^{(\text{post})} = M^{(\text{post})}(0,a) + M^{(\text{surf})}(a) \]
\[ M^{(\text{prior})}(a,\infty) = 0 \text{ (is included in breakup)} \]

**CDCC extension of R-matrix formalism**
- Simultaneous calculation of breakup and transfer cross sections
- Exterior term included in breakup, convergence issues removed
- More peripheral, reduce interior contribution
- Surface term dominant
Conclusions

Studying resonances with (d,p):
- Already underway at RIB facilities
- Conceptual and practical problems have to be overcome

**New formalism:**
- Builds on ideas from successful R-matrix approach
- Separation into interior and exterior regions works formally well, surface term emerges as important contributor, can be expressed in terms of familiar R-matrix parameters -> meaningful spectroscopic information
- Test cases show that the surface term is dominant; other contributions may not be negligible, but resonances less affected by interior contributions
- Including breakup via CDCC removes exterior prior contribution, thus eliminates convergence problem for resonances

Further studies will clarify conditions where the surface formalism will work well.

Promising approach for transfers to resonances.
TORUS Collaboration

ReactionTheory.org

TORUS: Theory of Reactions for Unstable iSotopes
A Topical Collaboration for Nuclear Theory

Theory of Reactions for Unstable iSotopes

A Topical Collaboration to develop new methods that will advance nuclear reaction theory for unstable isotopes by using three-body techniques to improve direct-reaction calculations and by developing a new partial-fusion theory to integrate descriptions of direct and compound-nucleus reactions. This multi-institution collaborative effort is directly relevant to three areas of interest identified in the solicitation: (b) properties of nuclei far from stability; (c) microscopic studies of nuclear input parameters for astrophysics and (e) microscopic nuclear reaction theory.

TORUS members

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Appendix
Investigating the role of the core-core interaction $V_{pA}$

3. $V_{pA}$ dependence of the various contributions

$$M = M^{(post)}(0,a) + M^{(surf)}(a) + M^{(prior)}(a, \infty)$$

This case:
- $^{90}\text{Zr}(d,p)$ for $E_d=11$ MeV
  - $^{91}\text{Zr}$ gs $(5/2^+)$
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Observations
- Overall cross section and relative strength of contributions varies with the strength of the core-core interaction
Investigating the role of the core-core interaction $V_{pA}$

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Observations
- Overall cross section and relative strength of contributions varies with the strength of the core-core interaction

Angular cross sections for $a=7$ fm

- 100% $V_{cc}$
- 90% $V_{cc}$
- 110% $V_{cc}$
Exploring R-matrix ideas for (d,p) one-nucleon transfers II

Transition matrix element $M$:
- Connects initial to final wave function
- Cross section $\sigma \sim M^2$

$M^{(\text{post})} = \langle \Phi_f^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} \rangle$

$\langle \varphi_F \chi_{pF}^{(-)} | \Delta V_{pF} | \varphi_d \varphi_A \chi_{dA}^{(+)} \rangle$

$\langle l_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \varphi_d \chi_{dA}^{(+)} \rangle$

$\Psi_i^{(+)}$ : exact d+A scattering function
$\Phi_f^{(-)} = \varphi_F \chi_{pF}^{(-)}$ exit channel function
$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$
$l_A^F = \langle \varphi_A | \varphi_F \rangle = l_A^F (r_{nA})$

$M^{(\text{prior})} = \langle \Psi_f^{(-)} | \Delta V_{dA} | \Phi_i^{(+)} \rangle$
$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$

One-body overlap of A and A+1 systems
- carries structure information
- typically approximated by single-particle function

3-body system

A
p
n
$\equiv$ CDCC

DWBA

3-body system

$\equiv$ CDCC

$\equiv$ CDCC
Assessing the R-matrix ideas - $^{48}$Ca

2. Surface contribution

$$M = M^{(\text{post})}(0,a) + M^{(\text{surf})}(a) + M^{(\text{prior})}(a,\infty)$$

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Peak cross section relative to full calculation

- $^{48}$Ca gs (3/2-)
- $^{49}$Ca 1$^{st}$ (1/2-)

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Graphs showing the contributions $M^{(\text{post})}(0,a)$, $M^{(\text{surf})}(a)$, and $M^{(\text{prior})}(a,\infty)$.