Coulomb distorted nuclear matrix elements in momentum space.

II. Computational aspects

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(d, p) reactions

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Effective Three-Body problem

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Faddeev equations with Coulomb interaction and nucleus excitation

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Optical Short Range (Nuclear) Potentials in Separable Form preferred

- Faddeev equations ⇒ preferrably solved in momentum space.
- (d, p) reaction with nucleus excitation ⇒ Separable Optical Short Range Potential.
- Coulomb interaction ⇒ switching to Coulomb distorted basis.

Required: Computational implementation of Separable Optical Potential in Coulomb distorted basis in momentum space.
Phenomenological optical potentials are usually in Woods-Saxon form in coordinate space.

**Example:** CH89 (central part)

\[
U_{\text{nucl}}(r) = V(r) + i(W(r) + W_s(r))
\]

**Separabilization:** generalized Ernst-Shakin-Thaler scheme for complex optical potentials.

Now the form factors are not the arbitrary functions, but half-shell \( t \)-matrices.

\[
U = \sum_{ij} u|\Psi_i^{(+)}\rangle \lambda \langle \Psi_j^{(-)}|u
\]

**Hint:** In/Out states are necessary to fulfill reciprocity theorem.
Quality of Separable Optical Potential: $l = 0$, $S$-matrix
Half-shell t-martix in Coulomb basis

For complex potentials, Coulomb distorted half-shell t-matrices (form factors) are not the complex conjugate of one another:

\[ u_{li}^{(C,1)}(p_\alpha) = \frac{1}{2\pi^2} \int dp \, p^2 \, u_{li}(p) \psi^C_{p_\alpha l}(p); \]
\[ u_{li}^{(C,2)}(p_\alpha) = \frac{1}{2\pi^2} \int dp \, p^2 \, u_{li}(p) [\psi^C_{p_\alpha l}(p)]^*; \]

\( \psi^C_{p_\alpha l}(p) \) is the half-shell Coulomb scattering wave function for the asymptotic momentum \( p_\alpha \):

\[ |\psi^C_{p_\alpha l}(p)\rangle = [1 + G_0(p_\alpha)T^C] |p\rangle. \]

- Special functions of complex arguments.
- Different representations for pole and non-pole regions.
- Gel’fand-Shilov regularization to deal with oscillatory singularity.
\[
\left\{ u^{(C,1)}_{li}(p_\alpha) = \frac{1}{2\pi^2} \int dp \, p^2 u_{li}(p) \psi_{lp_\alpha}^C(p) \right\}
\]

\( n + ^{208}\text{Pb} \) half-shell T-matrix (form factor) distorted as \( p + ^{208}\text{Pb} \)

\( L = 0; \) First term of rank 5

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\[
\left\{ u^{(C,1)}_{li}(p_\alpha) = \frac{1}{2\pi^2} \int dp \, p^2 \, u_{li}(p) \psi^C_{lp_\alpha}(p) \right\}
\]

\( p + ^{12}\text{C} \) half-shell short range T-matrices (form-factors)

\[ L = 0; \text{First term of rank 4} \]
Summary & Outlook

- **Faddeev-AGS framework in Coulomb basis passed the first test!**
  - Momentum space nuclear form-factors (half-shell T-matrices) obtained in a Coulomb distorted basis for high charges for the first time.
  - Ernst-Shakin-Thaler separabilization procedure successfully generalized for the case of complex optical potentials in momentum space. Realistic Separable (Generalized EST-type) Optical Potentials obtained for $n + ^{12}\text{C}$, $^{48}\text{Ca}$, $^{123}\text{Sn}$, and $^{208}\text{Pb}$ cases.
  - Algorithms to compute $\psi^C_{p\alpha l}(p)$ and the overlap integral successfully implemented for Generalized EST-type optical potentials. “Oscillatory singularity” of $\psi^C_{p\alpha l}(p)$ at $p = p_\alpha$ successfully regularized.

Near Future

Implementation of Faddeev-AGS equations in Coulomb basis to compute observables for $(d, p)$ reactions.
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Half-shell t-matrix: Difficulties to be addressed

\[ \psi_{p\alpha l}^C(p) = -\frac{4\pi}{\eta} e^{-\pi\eta/2} \Gamma(1 + i\eta) e^{i\alpha l} \left[ \frac{(p + p\alpha)^2}{4pp\alpha} \right]^l \times \text{Im} \left[ e^{-i\alpha l} \frac{(p + p\alpha + i0)^{-1+i\eta}}{(p - p\alpha + i0)^{1+i\eta}} \right] _2 F_1 \left( -l, -l - i\eta; 1 - i\eta; \frac{(p - p\alpha)^2}{(p + p\alpha)^2} \right) ; \]

\[ \eta = Z_1 Z_2 e^2 \mu/p\alpha. \]

- Computing special functions of complex arguments.
- Two different representations for pole and non-pole regions are required due to \( _2 F_1(a, b; c; z) \).
- \( \psi_{p\alpha l}^C(p) \) has ‘oscillatory singularity’ at \( p = p\alpha \).
  \[ \mapsto \text{Gel’fand-Shilov regularization} \text{ (reduce integrand around the pole, subtracting 2 terms of Taylor expansion)}. \]
Two representations of $\psi_{p\alpha l}^C(p)$

**Pole:**

$$
\psi_{p\alpha l}^C(p) = -\frac{4\pi}{p} e^{-\pi\eta/2} \Gamma(1 + i\eta) e^{i\alpha_l} \left[ \frac{(p + p\alpha)^2}{4pp\alpha} \right]^l \\
\times \text{Im} \left[ e^{-i\alpha_l} \frac{(p + p\alpha + i0)^{-1+i\eta}}{(p - p\alpha + i0)^{1+i\eta}} 2F_1 \left( -l, -l - i\eta; 1 - i\eta; \zeta \equiv \frac{(p - p\alpha)^2}{(p + p\alpha)^2} \right) \right].
$$

**Switching point:** $\zeta = \chi \approx 0.34$.

**Non-Pole:**

$$
\psi_{p\alpha l}^C(p) = -\frac{4\pi\eta e^{-\pi\eta/2} p\alpha (pp\alpha)^2}{(p^2 + p\alpha^2)^{l+1+i\eta}} \left[ \frac{\Gamma(l + 1 + i\eta) \Gamma(1/2)}{\Gamma(l + 3/2)} \right] \\
\times \left[ p^2 - (p\alpha + i0)^2 \right]^{-1+i\eta} 2F_1 \left( \frac{l + 2 + i\eta}{2}, \frac{l + 1 + i\eta}{2}; l + \frac{3}{2}; \chi \equiv \frac{4p^2p\alpha^2}{(p^2 + p\alpha^2)^2} \right).
$$

$\eta = Z_1 Z_2 e^2 \mu/p\alpha$. 

Gel’fand-Shilov regularization is the generalization of the Principal value regularization. The idea is to reduce the integrand $S(x)$ near the singularity$^1,^2$:

$$\int_{-\Delta}^{\Delta} dx \frac{\varphi(x)}{x^{1+i\eta}} = \int_{-\Delta}^{\Delta} dx \frac{\varphi(x) - \varphi(0) - \varphi'(0)x}{x^{1+i\eta}} - \frac{i\varphi(0)}{\eta} \left[\Delta^{-i\eta} - (\Delta)^{-i\eta}\right] + \ldots$$

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$^1$This formula is significantly simplified.

Pinch singularity and avoiding it

Pinch

\[ \psi_{lp\alpha}^C(k) \text{ has a singularity at } k = p\alpha. \text{ In general case of nuclear potential } V(p, p\alpha), \]

\[ (\psi_{lp}^C(k))^* \xrightarrow{k=p} V_{lpp\alpha}(k, \kappa) \xleftarrow{\kappa=p} \psi_{lp\alpha}^C(\kappa). \]  (1)

G. Cattappan et al. suggestion:
in case of separable potential, double integration procedure split onto two independent integrals, allowing to deal with this singularities separately, avoiding pinch\(^a\).