Applying the surface formalism for \((d,p)\) reactions to bound and resonance states

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Present theories provide valuable information on angular momenta …but have serious limitations in resonance cases

- Problem: present theories rely heavily on one-body overlap function of A and A+1 systems

\[ I_A^F(r) = < \phi_A | \phi_F > \]

- Carries structure information
- Not well-known in nuclear interior
- Typically approximated by single-particle function

- Calculations converge very slowly
- Not appropriate for describing reactions involving wide resonances
- Desired resonance properties (energies and widths) cannot be reliably obtained

Mukhamedzhanov’s suggestion: Extend R-matrix description to transfer reactions

Surface Formalism

Example: \(^{20}\text{O}(d,p)^{21}\text{O}\) inverse-kinematics experiment - Interpreted the traditional way

Fernandez-Dominguez et al, PRC 84, 011301(R) (2011)
The surface formalism – DWBA and CDCC approximations

Transition matrix element $M$:
- Connects initial to final wave function
- Cross section $\sigma \sim M^2$

**Reminder: (d,p) formalism**

**The surface formalism – DWBA and CDCC approximations**

$M^{(\text{post})} = \langle \Phi_f^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} \rangle$

**DWBA**

$\langle \Phi_F \chi_{pF}^{(-)} | \Delta V_{pF} | \Phi_d \Phi_A \chi_{dA}^{(+)} \rangle$

$\langle I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \Phi_d \chi_{dA}^{(+)} \rangle$

$\Psi_i^{(+)}$: exact d+A scattering function

$\Phi_f^{(-)} = \Phi_F \chi_{pF}^{(-)}$ exit channel function

$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$

$I_A^F = \langle \Phi_A | \Phi_F \rangle$ one-body overlap

**3-body**

$\langle \Phi_F \chi_{pF}^{(-)} | \Delta V_{pF} | \Phi_A \Psi_i^{3B(+)} \rangle$

$\langle I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \Psi_i^{3B(+)} \rangle$

**CDCC**

$\langle I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \Psi_i^{\text{CDCC}(+)} \rangle$

$M^{(\text{prior})} = \langle \Psi_f^{(-)} | \Delta V_{dA} | \Phi_i^{(+)} \rangle$

$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$
Some details of the surface formalism

Reminder: surface formalism

Transition matrix element $M$:
- Cross section $\sigma \sim M^2$
- Is split into interior and exterior parts

$$M^{(\text{post})} = \langle \Phi_f^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} \rangle$$

**DWBA**

$$\langle I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \phi_d \chi_{dA}^{(+)} \rangle$$

Post:
$$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$$

Prior:
$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

**Interior + exterior**

$$M^{(\text{post})} = M^{(\text{post})}(0,a) + M^{(\text{post})}(a,\infty)$$

$$M^{\text{post}} = M^{\text{surf}}(a) + M^{\text{prior}}(a,\infty)$$

**Surface formulation**

$$M = M^{(\text{post})}(0,a) + M^{\text{surf}}(a) + M^{\text{prior}}(a,\infty)$$

One-body overlap function of A and A+1 systems

3-body system
The surface formalism: a closer look

Features:
• Model dependence contained in interior term
• Surface term can be written in terms of reduced widths, the surface radius, and derivatives of known functions.
• Exterior term vanishes in CDCC implementation

Questions:
• Is the surface term dominant? Where?
• Do we get reduced dependence on model for interior? Under which circumstances?
• Can we extract useful spectroscopic quantities from comparison to experiment?

Surface formulation
\[ M = M^{(post)}(0,a) + M_{surf}(a) + M^{(prior)}(a,\infty) \]

- **model dependence**
- **asymptotic quantities**

\[ M_{surf}(a) = f(a, C_A^F, B_{nA}) \]
\[ B_{nA} = \log \text{derivative of } I_A^F \text{ at surface radius } a \]
ANC: \( C_A^F \) defined through: \( I_A^F \left( r_{nA} \right) \rightarrow C_A^F W(k r_{nA}) \)
related to reduced width amplitude \( C_A^F \sim \gamma_{nA} \)

Tests completed:
- \(^{90}\text{Zr}(d,p)\) for \( E_d=11 \) MeV
  - \(^{91}\text{Zr}\) gs, 1st excited state, \( 2f_{7/2} \) resonance
- \(^{48}\text{Ca}(d,p)\) for \( E_d=13, 19.3, 56 \) MeV
  - \(^{49}\text{Ca}\) gs, 1st excited state
- \(^{20}\text{O}(d,p)\) for \( E_d=21 \) MeV
  - \(^{21}\text{O}\) gs, 1st excited state, \( 1d_{3/2} \) and \( 1f_{7/2} \) resonances
- Akram:
  - \(^{12}\text{C}(d,p)\) for \( E_d=30 \) MeV
  - \(^{40}\text{Ca}(d,p)\) for \( E_d=34.4 \) MeV
  - \(^{209}\text{Pb}(d,p)\) for \( E_d=52 \) MeV
Internal, surface, external contributions – $^{90}$Zr(d,p) at $E_d=11$ MeV

$$M = M^{(\text{post})}(0,a) + M^{(\text{surf})}(a) + M^{(\text{prior})}(a,\infty)$$

**Observations**
- Surface term dominant at 6-8 fm
- Small interior contributions
- Small exterior contributions
- Surface term does not produce the whole cross section

The surface term is dominant, but contributions from the interior and exterior terms remain.

Escher, Thompson, Mukhamedzhanov, JPCS (2012).
The surface contribution – $^{90}$Zr(d,p) at $E_d=11$ MeV

- Cross sections depend on surface radius
- The surface term is dominant, but corrections remain
Effect of varying the beam energy – $^{48}\text{Ca}(d,p)$ at $E_d=13, 19.3, 56 \text{ MeV}$

Angular cross section – Surface term only

Peak cross section relative to full calculation

Surface term approximation improves with decreasing energy

Calculations for $^{49}\text{Ca} \text{ 1st} (1/2^-)$ give similar results
The oxygen case - $^{20}\text{O}$ at $E_d=21$ MeV

Angular cross section – Surface term only

Peak cross section relative to full calculation

Surface at 5 fm approximately reproduces measurement.

$^{20}\text{O}$ gs (5/2+)

$^{20}\text{O}$ gs (1/2+)
Resonances – $^{90}\text{Zr}$ at $E_d=11$ MeV

- Results similar to bound-state cases
- Surface term dominant at larger radii
- Interior/exterior terms still contribute
Resonances - $^{20}$O at $E_d=21$ MeV

Angular cross section – Surface term only

Peak cross section relative to full calculation

• Calculations reasonable only for $a<7$ fm
• Convergence difficult for resonance cases
• Surface term seems able to reproduce data

$^{21}$O $d_{3/2}$ resonance at 0.964 MeV
$^{21}$O $d_{3/2}$ resonance at 2.364 MeV
$^{21}$O $f_{7/2}$ resonance at 2.364 MeV
Lessons so far…

• Varying the surface radius changes the relative contributions from interior, surface, exterior terms.
• The surface term is dominant in the surface region, but contributions from the interior and/or exterior terms are present at all radii.
• The surface term can provide a rough approximation to the (d,p) cross section. The approximation deteriorates for higher beam energies.
• The findings are similar for all mass regions considered.
• Results for resonances are similar to those for bound states.
• Achieving convergence for resonances is difficult, but expected to be simpler in the fully-implemented method.
Maximizing the surface term...

- Motivation: The surface term can be written in terms of reduced widths, the surface radius, and derivatives of known functions.

- The calculations suggest: using a slightly smaller radius and the CDCC implementation (which eliminates the ext-prior contribution) → Ian’s talk

- AMM: Vary the core-core potential to simultaneously minimize the 2^{nd}-order contributions and the interior-post term → Not a solution (formally or practically)!

\[ M^{(\text{post})} = \langle \Phi_f(-) | \Delta V_{pF} | \Psi_i(+) \rangle \]
\[ \Delta V_{pF} = V_{pA} + V_{pn} - U_{pF} \]
Lessons for moving forward...

To make the surface approach into a useful tool, we need to:

• Implement the formalism in its CDCC form, to incorporate breakup and eliminate the exterior-prior contributions → Ian’s talk

• Minimize the interior-post contributions by finding an optimal radius (corrections may still be necessary)

• Test the approach for bound and resonance states
Appendix
The surface contribution – $^{90}\text{Zr}(d,p)$ at $E_d=11$ MeV

Cross section contributions from the three terms

Peak cross section relative to full calculation

- Interior and exterior terms still contribute to the cross section

$^{91}\text{Zr \, gs \, (5/2+)}$

$^{91}\text{Zr \, 1st \, (1/2+)}$

- Interior and exterior terms still contribute to the cross section
The surface formalism: a closer look - II

\[ M = M^{(\text{post})}(0,a) + M_{(\text{surf})}(a) + M^{(\text{prior})}(a,\infty) \]

- **Model dependence**
- **Asymptotic quantities**

\[
M_{(\text{surf})}(a) = \sqrt{\frac{R_{nA}}{2\mu_{nA}}} \sum_{J_{nA}M_{nA}j_{nA}m_{nA}} \langle J_AM_A j_{nA}m_{nA} | J_FM_F \rangle \langle J_nM_n l_{nA}m_{nA} | j_{nA}m_{j_{nA}} \rangle \langle J_pM_p J_nM_n | J_dM_d \rangle \gamma_{nAJ_{nA}l_{nA}} \]

\[
\times \int dr_{pF} \gamma_{-k_{pF}}^{(+)}(r_{pF}) \int d\Omega_{r_{nA}} Y_{l_{nA}m_{nA}}^{*} (\hat{r}_{nA}) \left[ \varphi_{d}(r_{pn}) \gamma_{k_{dA}}^{(+)}(r_{dA}) (B_{nA} - 1) - \frac{\partial \varphi_{d}(r_{pn}) \gamma_{k_{dA}}^{(+)}(r_{dA})}{\partial r_{nA}} \right] \bigg|_{r_{nA} = \hat{r}_{nA}}
\]

From: Mukhamedzhanov, PRC 84, 044616 (2011)
Extension of the formalism to include breakup

**DWBA matrix element**

\[ M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty) \]

**CDCC matrix element**

\[ M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a) \]
\[ M^{(prior)}(a,\infty) = 0 \text{ (is included in breakup)} \]

**CDCC (Continuum-discretized coupled channels)**
- Approximate treatment of 3-body problem
- Describes breakup of deuteron
- Successfully used for describing data
- Currently revisited via comparison with Fadeev

**CDCC extension of R-matrix formalism**
- Simultaneous calculation of breakup and transfer cross sections
- Exterior term included in breakup, convergence issues removed
- More peripheral, reduce interior contribution
- Surface term dominant