The ‘half moon’ singularity
Discussion at annual TORUS Collaboration meeting

Vasily Eremenko

1Institute for Nuclear & Particle Physics and Dept. of Physics & Astronomy, Ohio University

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Introduction of Faddeev equations.
What is the ‘half moon’ singularity?
Traditional ways to deal with this singularity.
New outlook on the ‘half moon’ problem.
\[ T|\phi\rangle = tP|\phi\rangle + tPG_0T|\phi\rangle. \]

Amplitude is\(^a\)

\[ \langle \vec{p}\vec{q}|U_0|\phi\rangle = \langle \vec{p}\vec{q}|(1 + P)T|\phi\rangle. \]

\(^a|\phi\rangle = |\varphi_d\bar{q}_0\rangle.\]
What is the ‘half moon’ singularity?

Expression with explicit singularity (3D)

\[ G_0 = \frac{1}{E + i\epsilon - \frac{1}{m} \left( p''^2 + \frac{3}{4} q''^2 \right)} , \]

\[ \langle p' q | p'' q'' \rangle = \delta(p' + \bar{\pi}_1) \delta(p'' - \bar{\pi}_2) + \delta(p' - \bar{\pi}_1) \delta(p'' + \bar{\pi}_2) , \]

\[ \langle \bar{p} q | tP | \varphi_d \bar{q}_0 \rangle = T_0(\bar{p}, \bar{q}; \bar{q}_0) , \quad \bar{\pi}_1 = \frac{1}{2} \bar{q} + \bar{q}'', \quad \bar{\pi}_2 = \bar{q} + \frac{1}{2} \bar{q}'' . \]

\[ t_s(\bar{p}, \bar{\pi}_1; z) = t(\bar{p}, \bar{\pi}_1; z) + t(\bar{p}, -\bar{\pi}_1; z) . \]

\[ T(\bar{p}, \bar{q}; \bar{q}_0) = T_0(\bar{p}, \bar{q}; \bar{q}_0) + \int d^3 \bar{q}'' \frac{t_s(\bar{p}, -\bar{\pi}_1; E - \frac{3}{4m} q^2) \ T(\bar{\pi}_2, \bar{q}''; \bar{q}_0)}{E + i\epsilon - \frac{1}{m} \left( q^2 + q''^2 + \bar{q} \cdot \bar{q}'' \right)} . \]
What is the ‘half moon’ singularity?

After partial wave decomposition

\[ \cdots \int dq'' \int_{-1}^{1} dx \frac{1}{E + i\epsilon - \frac{1}{m} (q^2 + q''^2 + qq''x)}. \]

Integration over \( x \)

\[ \forall q < q_{max}, \ \exists \{q'', x_0\} : \ q^2 + q''^2 + qq''x_0 = mE, \Rightarrow \text{singularity}. \]

\[ \cdots \Rightarrow \ln \left| \frac{1 + x_0}{1 - x_0} \right|. \]

Sometimes this singularity is located just at the end of the integration region \((x_0 = \pm 1)\).

\[ q_{max} = \sqrt{\frac{4m}{3}}E, \quad x_0 = \frac{mE - q^2 - q''^2}{qq''}. \]
What is the ‘half moon’ singularity?

Illustration

\[ x_0(q_+) = -1 \]

\[ |x_0| \leq 1 \]

\[ x_0(q_-) = -\frac{q}{|q_+|} \]
Traditional ways to deal with this singularity

- Switch to the complex plain of $q''$ (Hetherington & Schick). No way! We don’t have an analytic continuation of our functions to the complex plain.
- Subtraction (W. Glöckle & H. Witała).
- Using splines (W. Glöckle, Ch. Elster, & H. Liu):

$$f(y) \Rightarrow a_0 + a_1 y + a_2 y^2 + a_3 y^3 \Rightarrow \text{semi-analytical integration.}$$
W. Glöckle’s suggestion

References


The idea
Detangle $q$ and $q''$ in the denominator using the internal $\delta$-functions.

Example:

$$\delta(p' - \pi_1) = \frac{2p'}{qq''} \delta(x - X) \Theta(1 - |X|), \quad X = \frac{p'{}^2 - \frac{1}{4}q^2 - q''^2}{qq''}.$$
The Results:

\[ \cdots \int dq'' q''^2 \frac{\cdots \Theta(\cdots) \cdots}{E + i \epsilon - \frac{3}{4m} q''^2 - E_d} + \cdots \int dp' p'^2 \frac{\cdots \Theta(\cdots) \cdots}{E + i \epsilon - \frac{1}{m} (p'^2 + \frac{3}{4} q^2)} .\]

Due to \( \Theta \)-functions, integration only outside of the rectangular region.
No coupled singularities!